

# Spin-orbit coupling in Wien2k

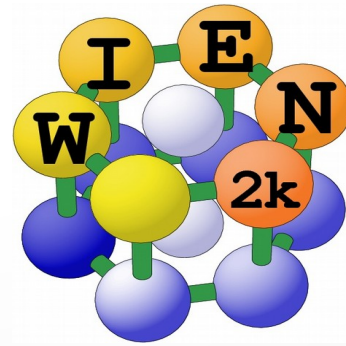
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Agency for  
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# Dirac Hamiltonian

Quantum mechanical description of electrons, consistent with the theory of special relativity.

$$H_D = c \vec{\alpha} \cdot \vec{p} + \beta m c^2 + V$$

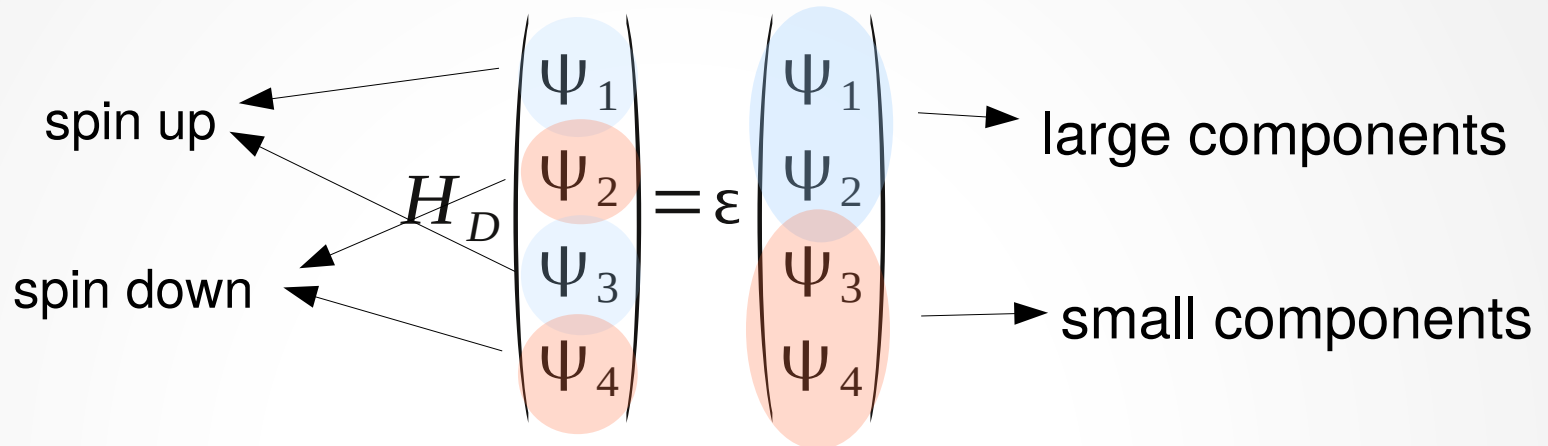
$$\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix} \quad \beta_k = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}$$

Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$
$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$H_D$  and the wave function are 4-dimensional objects

# Dirac Hamiltonian



free particle:

$$\begin{pmatrix} \epsilon - mc^2 & 0 & -\hat{p}_z & -(\hat{p}_x - i\hat{p}_y) \\ 0 & \epsilon - mc^2 & -(\hat{p}_x + i\hat{p}_y) & \hat{p}_z \\ -\hat{p}_z & -(\hat{p}_x - i\hat{p}_y) & \epsilon + mc^2 & 0 \\ -(\hat{p}_x + i\hat{p}_y) & \hat{p}_z & 0 & \epsilon + mc^2 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = 0$$

slow particle limit  
( $p=0$ ):

$$mc^2, \begin{pmatrix} \psi \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad mc^2, \begin{pmatrix} 0 \\ \psi \\ 0 \\ 0 \end{pmatrix}$$

spin up    spin down

$$-mc^2, \begin{pmatrix} 0 \\ 0 \\ \psi \\ 0 \end{pmatrix} \quad -mc^2, \begin{pmatrix} 0 \\ 0 \\ 0 \\ \psi \end{pmatrix}$$

antiparticles, up, down

# Dirac equation in spherical potential

## Solution for spherical potential

$$\Psi = \begin{pmatrix} g_{\kappa}(r) \chi_{\kappa\sigma} \\ -i f_{\kappa}(r) \chi_{\kappa\sigma} \end{pmatrix} \rightarrow \text{combination of spherical harmonics and spinors}$$

$$\begin{aligned} \kappa &= -s(j+1/2) \\ j &= l+s/2 \\ s &= +1, -1 \end{aligned}$$



$$\begin{aligned} \frac{dg_{\kappa}}{dr} &= -\frac{(\kappa+1)}{r} g_{\kappa} + 2Mc f_{\kappa} \\ \frac{df_{\kappa}}{dr} &= \frac{1}{c} (V - E) g_{\kappa} + \frac{\kappa-1}{r} f_{\kappa} \end{aligned}$$

Radial Dirac equation

# Dirac equation in spherical potential

Radial Dirac equation

$$\frac{dg_{\kappa}}{dr} = -\frac{(\kappa+1)}{r} g_{\kappa} + 2 M c f_{\kappa}$$

$$\frac{df_{\kappa}}{dr} = \frac{1}{c} (V - E) g_{\kappa} + \frac{\kappa-1}{r} f_{\kappa}$$

$\kappa$  dependent term, for a constant  $l$ ,  $\kappa$  depends on the sign of  $s$

substitute  $f$  from first eq. into the second eq.

$$-\frac{1}{2M} \left[ \frac{d^2 g_{\kappa}}{dr^2} + \frac{2}{r} \frac{dg_{\kappa}}{dr} - \frac{l(l+1)}{r^2} g_{\kappa} \right] - \frac{dV}{dr} \frac{dg_{\kappa}}{dr} - \frac{1}{4M^2 c^2} + V g_{\kappa} - \frac{\kappa-1}{r} \frac{dV}{dr} \frac{g_{\kappa}}{4M^2 c^2} = E g_{\kappa}$$

scalar relativistic approximation

spin-orbit coupling

# Implementation: core electrons

**Core states** are calculated with spin-compensated Dirac equation

For **spin polarized potential** – spin up and spin down radial functions are calculated separately, the density is averaged according to the occupation number specified in *case.inc* file

Relations between quantum numbers

		<b><math>j=l+s/2</math></b>		<b><math>\kappa=-s(j+1/2)</math></b>		<b>occupation</b>	
	<b><math>l</math></b>	<b><math>s=-1</math></b>	<b><math>s=+1</math></b>	<b><math>s=-1</math></b>	<b><math>s=+1</math></b>	<b><math>s=-1</math></b>	<b><math>s=+1</math></b>
<b>s</b>	<b>0</b>		<b>1/2</b>		<b>-1</b>		<b>2</b>
<b>p</b>	<b>1</b>	<b>1/2</b>	<b>3/2</b>	<b>1</b>	<b>-2</b>	<b>2</b>	<b>4</b>
<b>d</b>	<b>2</b>	<b>3/2</b>	<b>5/2</b>	<b>2</b>	<b>-3</b>	<b>4</b>	<b>6</b>
<b>f</b>	<b>3</b>	<b>5/2</b>	<b>7/2</b>	<b>3</b>	<b>-4</b>	<b>6</b>	<b>8</b>

$1s^{1/2}$  ← 9 0.00  
 ( 1, -1, 2 ) ( N, KAPPA, OCCUP )  
 ( 2, -1, 2 ) ( N, KAPPA, OCCUP )  
 $2p^{1/2}$  ← ( 2, 1, 2 ) ( N, KAPPA, OCCUP )  
 ( 2, -2, 4 ) ( N, KAPPA, OCCUP )  
 $2p^{3/2}$  ← ( 3, -1, 2 ) ( N, KAPPA, OCCUP )  
 ( 3, 1, 2 ) ( N, KAPPA, OCCUP )  
 ( 3, -2, 4 ) ( N, KAPPA, OCCUP )  
 ( 3, 2, 4 ) ( N, KAPPA, OCCUP )  
 ( 3, -3, 6 ) ( N, KAPPA, OCCUP )

Core levels configuration  
(*case.inc* for Ru atom)<sub>6</sub>

# Implementation: valence electrons

Valence electrons **inside atomic spheres** are treated within **scalar relativistic approximation** (Koelling and Harmon, *J. Phys C* 1977) if **RELA** is specified in *struct* file

$$\frac{dP}{dr} - \frac{1}{r} P = 2McQ$$

$$\frac{dQ}{dr} - \frac{1}{r} Q = \left[ l \frac{(l+1)}{2} Mcr^2 + \frac{(V-\epsilon)}{c} \right] P$$

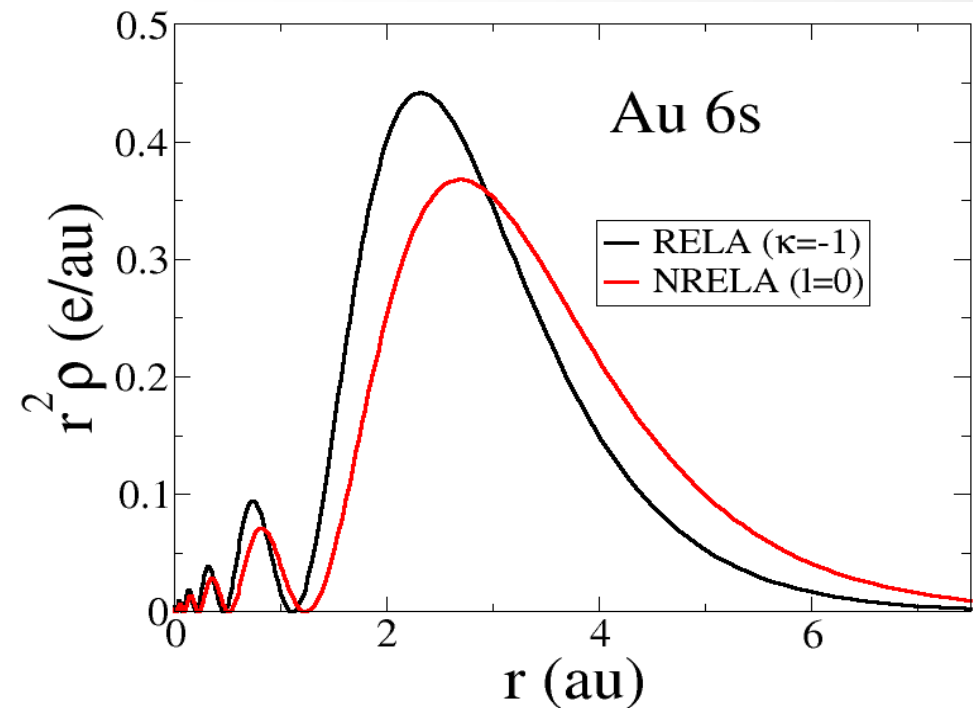
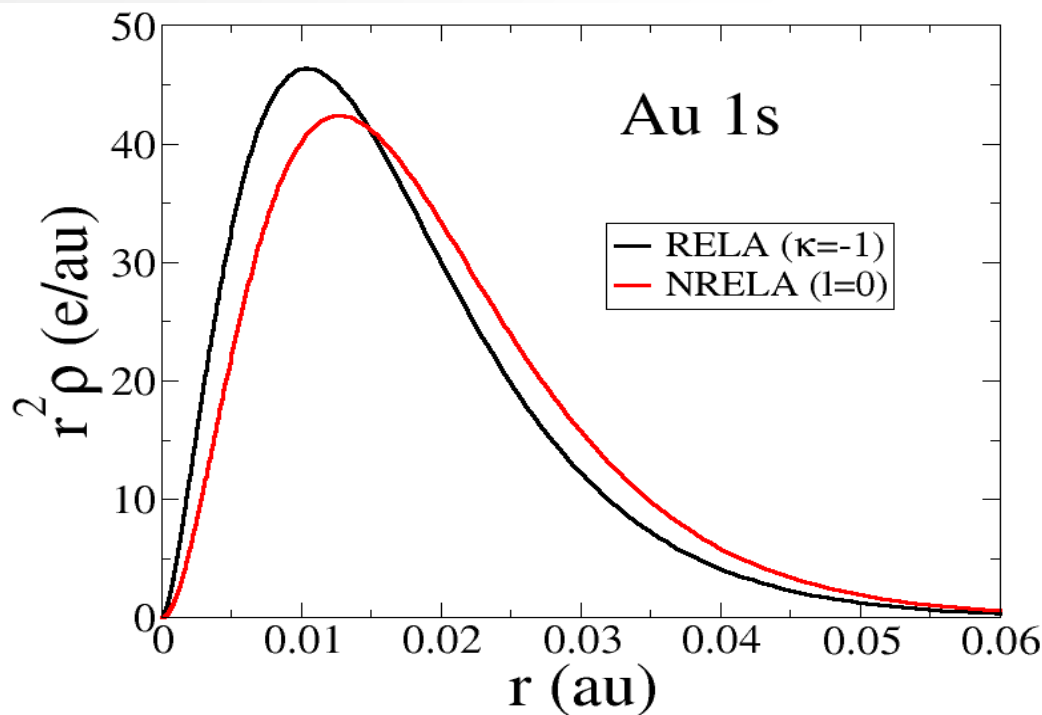
radial equations of Koelling and Harmon (**spherical potential**)

- no  $\kappa$  dependency of the wave function, (l,m,s) are good quantum numbers
- all relativistic effects are included except SOC
- small component enters normalization and calculation of charge inside spheres
- augmentation with large component only
- SOC can be included in “second variation”

Valence electrons in **interstitial** region are non-relativistic

# Effects of *RELA*

- contraction of Au s orbitals



- 1s contracts due to relativistic mass enhancement
- 2s - 6s contract due to orthogonality to 1s

$$M V^2 / r = Z e / r^2$$

centripetal force

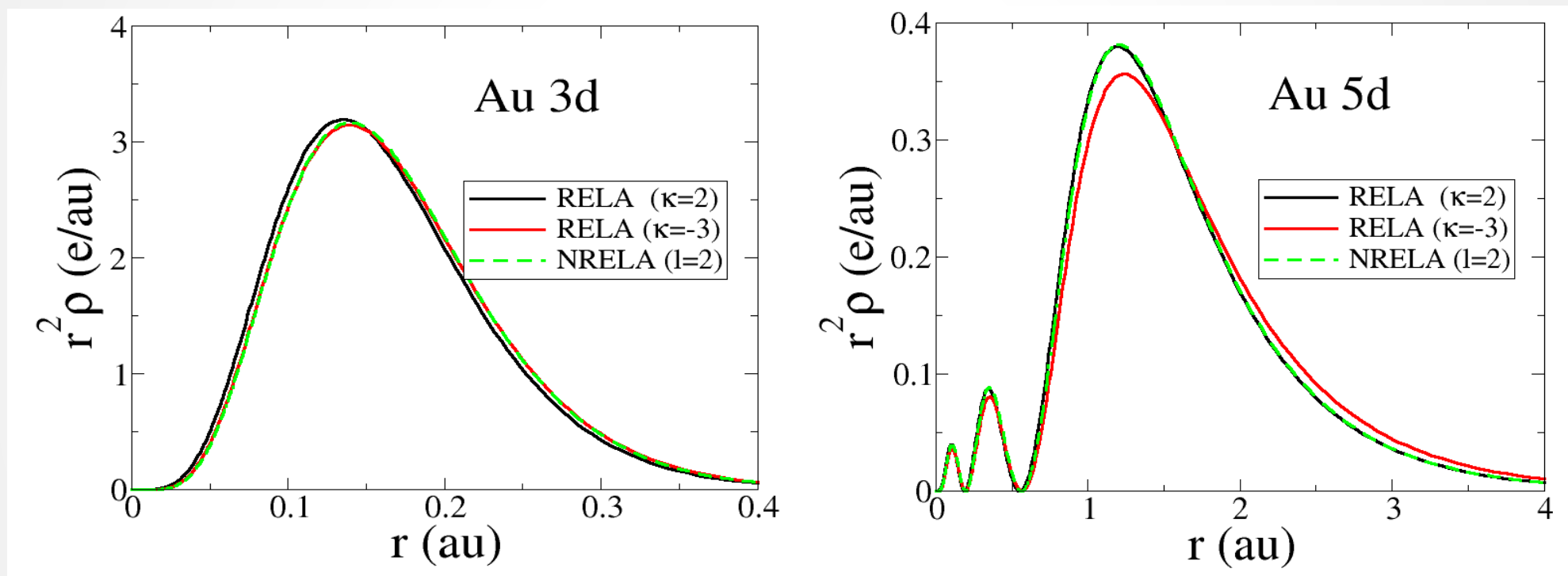
$$M = m / \sqrt{1 - (v/c)^2}$$

$v \sim Z$ : Au  $Z = 79$ ;  $M = 1.2 m$



# Effects of *RELA*

orbital expansion of Au d orbitals



Higher  $l$ -quantum number states **expand due to better shielding of the core charge** from contracted s-states (effect is larger for higher states).

# Spin orbit-coupling

$$H_P = -\frac{\hbar}{2m} \nabla^2 + V_{ef} + \zeta (\vec{\sigma} \cdot \vec{l}) \dots \quad \zeta = \frac{1}{2Mc^2} \frac{1}{r^2} \frac{dV_{MT}(r)}{dr}$$

- 2x2 matrix in spin space, due to Pauli spin operators, wave function is a 2-component vector (spinor)

$$H_P \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \epsilon \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

spin up ←
→ spin down

Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Spin structure of the Hamiltonian with SOC

$$\left( \begin{array}{cc} -\frac{\hbar}{2m} \nabla^2 + V_{ef} & 0 \\ 0 & -\frac{\hbar}{2m} \nabla^2 + V_{ef} \end{array} + \begin{pmatrix} \zeta l_z + \dots & \zeta (l_x - i l_y) \\ \zeta (l_x + i l_y) & -\zeta l_z + \dots \end{pmatrix} \right) \Psi = \epsilon \Psi$$

# Spin orbit-coupling

- SOC is active only **inside atomic spheres**, only **spherical potential** ( $V_{MT}$ ) is taken into account, in the polarized case spin up and down parts are averaged
- **eigenstates are not pure spin states**
- **off-diagonal term of the spin density matrix do not enter SCF cycle**
- SOC is added in a **second variation** (*lapwso*):

first diagonalization (lapw1)

$$H_1 \psi_1 = \varepsilon_1 \psi_1$$

second diagonalization (lapwso)

$$(H_1 + H_{SO}) \psi = \varepsilon \psi$$

second diagonalization

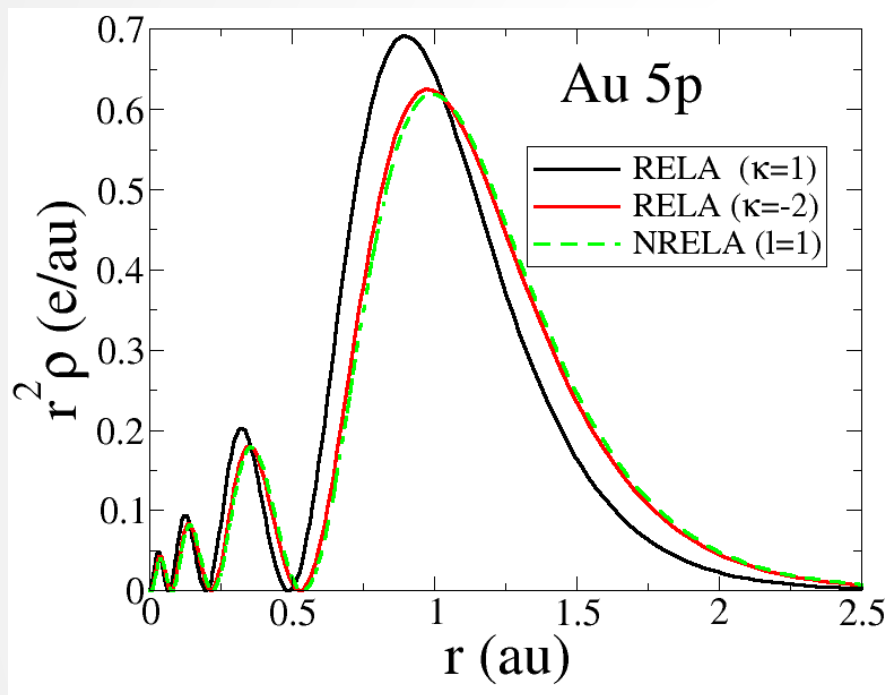
$$\sum_i^N \left( \delta_{ij} \varepsilon_1^j + \langle \psi_1^j | H_{SO} | \psi_1^i \rangle \right) \langle \psi_1^i | \psi \rangle = \varepsilon \langle \psi_1^j | \psi \rangle$$

sum includes both up/down spin states

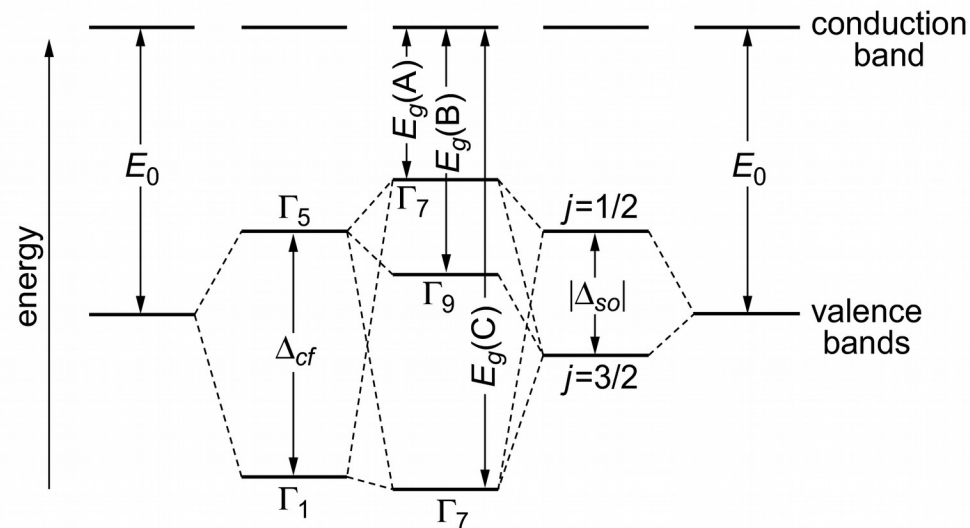
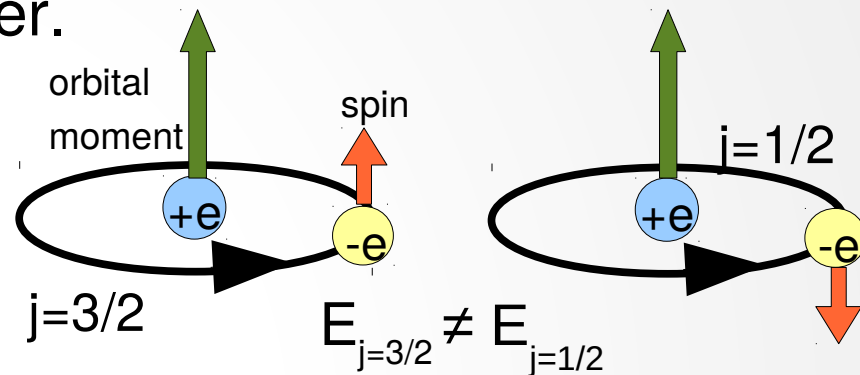
**N is much smaller than the basis size in lapw1!!**

# SOC splitting of p states

Spin Orbit splitting of l-quantum number.



$p_{1/2}$  ( $\kappa=1$ ) different behavior than non-relativistic p-state (density is diverging at nucleus), thus there is a need for extra basis function ( $p_{1/2}$  orbital)

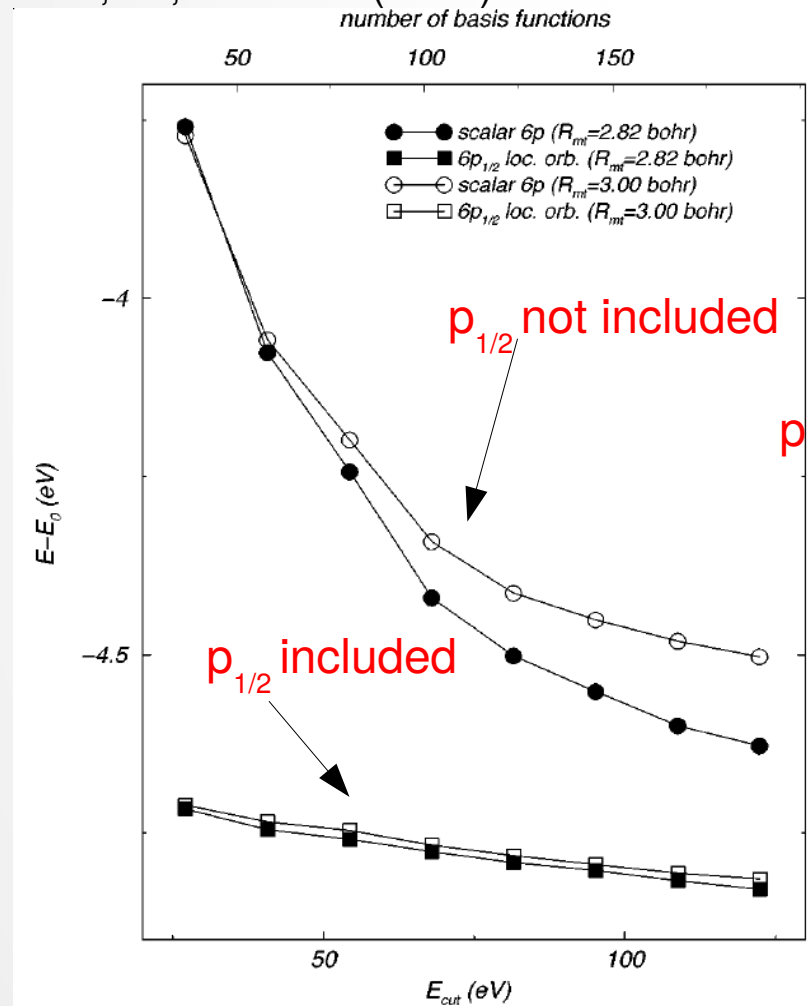


band edge at  $\Gamma$  in ZnO

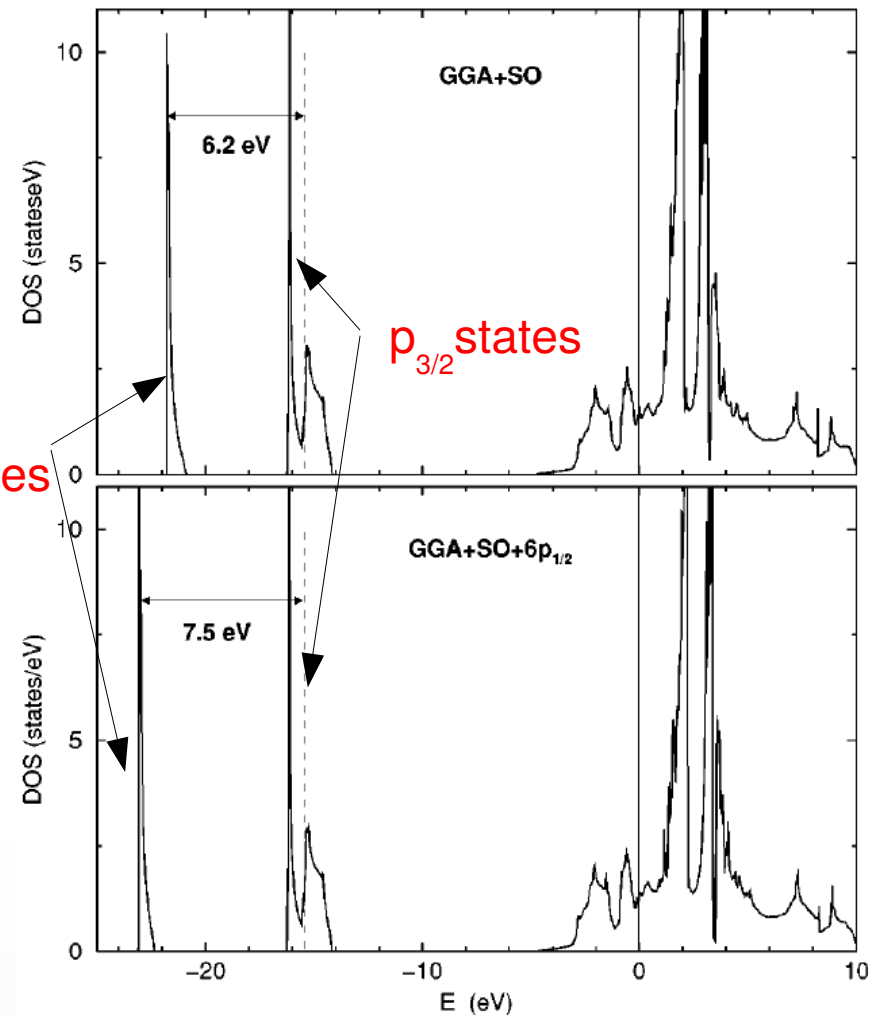
# $p_{1/2}$ orbitals

## Electronic structure of fcc Th, SOC with $6p_{1/2}$ local orbital

PRB, 64, 1503102 (2001)

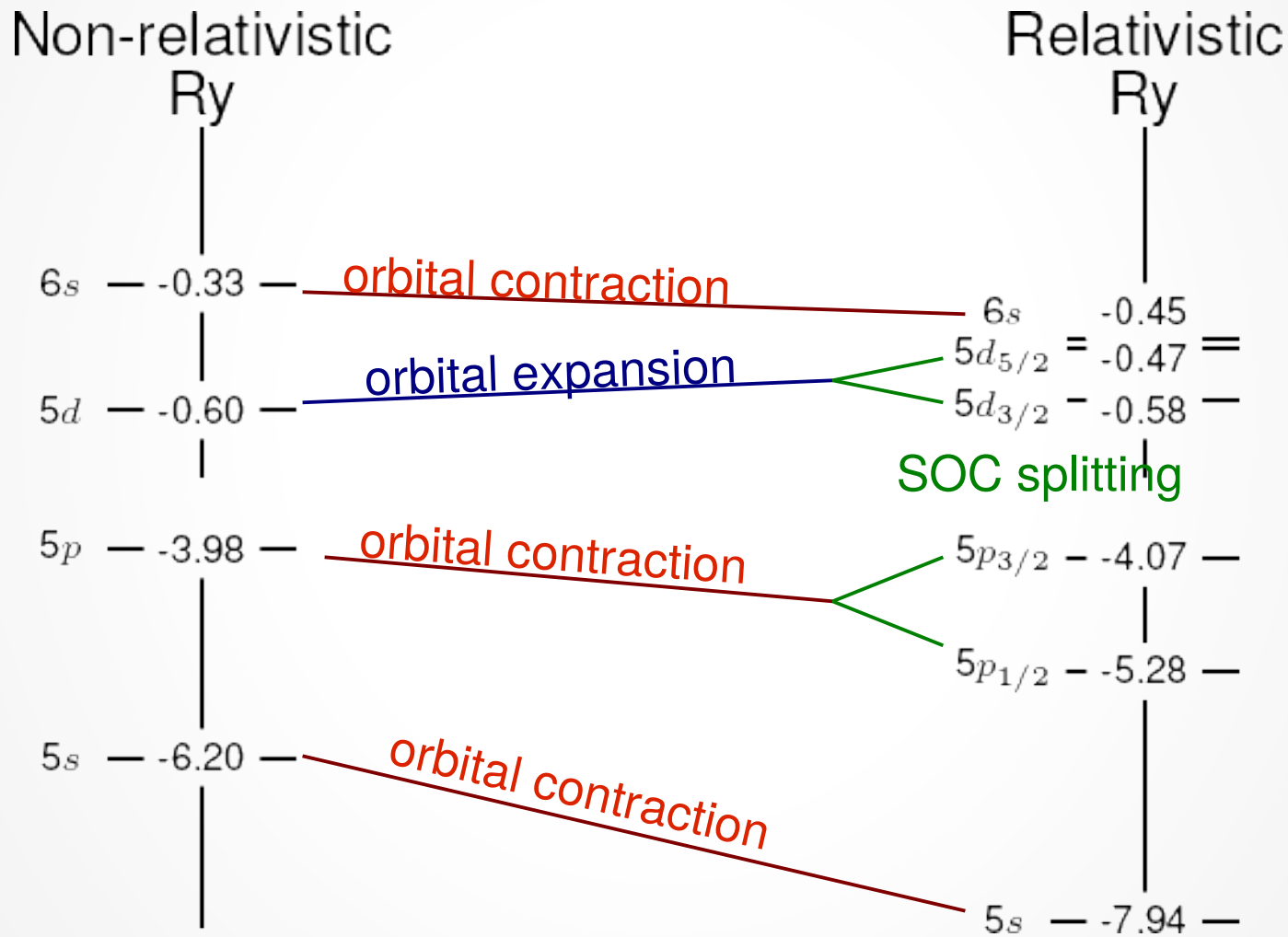


energy vs. basis size



DOS with and without  $p_{1/2}$

# Au atomic spectra

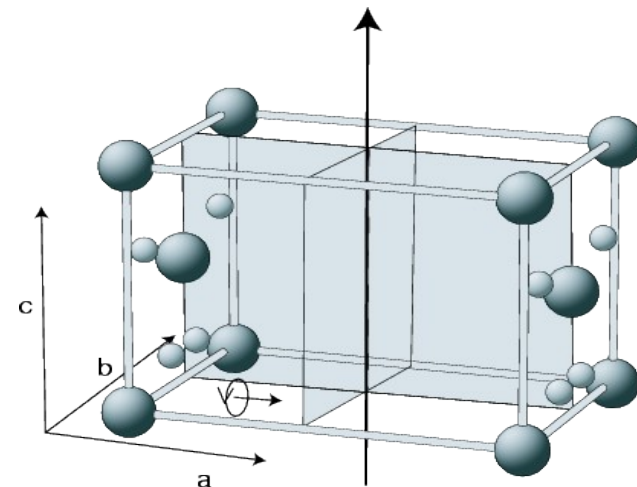


# SOC in magnetic systems

- **SOC couples magnetic moment to the lattice**
  - direction of the exchange field matters (input in case.inso)
- **symmetry operations acts in real and spin space**
  - number of symmetry operations may be reduced
  - no *time* inversion
  - *initso\_lapw* (must be executed) detects new symmetry setting

direction of magnetization

	[100]	[010]	[001]	[110]
1	A	A	A	A
$m_a$	A	B	B	-
$m_b$	B	A	B	-
$2_c$	B	B	A	B



# SOC in Wien2k

- run(sp)\_lapw -so script:

**x** *lapw1*

(increase E-max for more eigenvectors in second diag.)

**x** *lapwso*

(second diagonalization)

**x** *lapw2 -so*

(SOC **ALWAYS** needs complex lapw2 version)

case.inso file:

WFFIL

4 1 0

-10.0000 1.50000

0. 0. 1.

1

2 -0.97 0.005

0 0 0 0 0

lmax,ipr,kpot

emin,emax (output energy window)

direction of magnetization (lattice vectors)

number of atoms for which RLO is added

atom number,e-lo,de (case.in1), repeat NX times

number of atoms for which SO is switched off; list of atoms

$p_{1/2}$  orbitals, **use with caution !!**

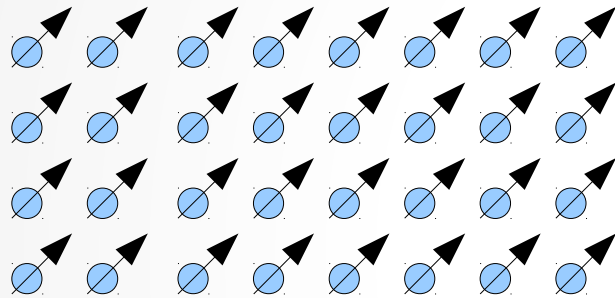


# Summary: relativistic effects

- **core electrons** - Dirac equation using spherical part of the total potential (dirty trick for spin polarized systems)
- **valence electrons** - scalar relativistic approximation is used as default (RELA switch in *case.struct*),
- SOC for **valence electrons** - *lapwso* has to be included in SCF cycle (*run -so/run\_sp -so*), atomic spheres only
- limitations: **not all programs are compatible with SOC**, for instance: no forces with SOC (yet)

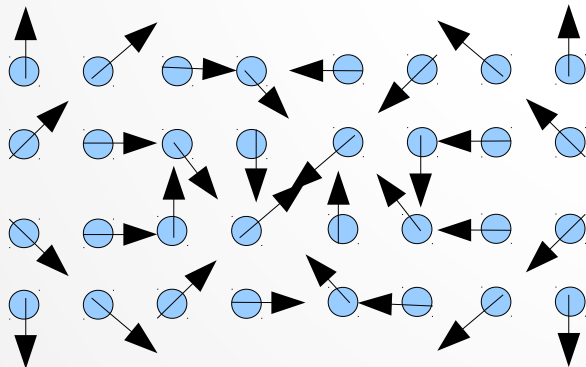
# magnetism, non-collinear case

- WIEN2k can do only **nonmagnetic** or **collinear** magnetic structures



$$\psi_{\uparrow} = \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix}, \quad \psi_{\downarrow} = \begin{pmatrix} 0 \\ \psi_2 \end{pmatrix}$$

- noncollinear** magnetic structures, use WIENNCM



$$\psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}, \quad \Psi_1, \Psi_2 \neq 0$$

# Pauli Hamiltonian

$$H_P = -\frac{\hbar^2}{2m} \nabla^2 + V_{ef} + \mu_B \vec{\sigma} \cdot \vec{B}_{ef} + \zeta (\vec{\sigma} \cdot \vec{l}) \dots$$

- 2x2 matrix in spin space, due to Pauli spin operators
- wave function is a 2-component vector (spinor)

$$H_P \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \epsilon \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

spin up component

spin down component

Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

# Pauli Hamiltonian

$$H_P = -\frac{\hbar}{2m} \nabla^2 + V_{ef} + \mu_B \vec{\sigma} \cdot \vec{B}_{ef} + \zeta (\vec{\sigma} \cdot \vec{l}) \dots$$

electrostatic potential
magnetic field
spin-orbit coup.

$$V_{ef} = V_{ext} + V_H + V_{xc} \qquad B_{ef} = B_{ext} + B_{xc}$$

Hartee term
exchange-correlation potential
exchange-correlation field

- exchange-correlation potential  $V_{xc}$  and magnetic field  $B_{xc}$  are defined within DFT LDA or GGA

# Exchange and correlation

- from DFT LDA exchange-correlation energy:

$$E_{xc}(n, \vec{m}) = \int n \epsilon_{xc}(n, \vec{m}) dr^3$$

local function of n and m

- definition of  $V_{cx}$  and  $B_{xc}$ :

$$V_{xc} = \frac{\partial E_{xc}(n, \vec{m})}{\partial n} \quad \vec{B}_{xc} = \frac{\partial E_{xc}(n, \vec{m})}{\partial \vec{m}}$$

functional derivatives

- LDA expression for  $V_{cx}$  and  $B_{xc}$ :

$$V_{xc} = \epsilon_{xc}(n, \vec{m}) + n \frac{\partial \epsilon_{xc}(n, \vec{m})}{\partial n}$$

$$\vec{B}_{xc} = n \frac{\partial \epsilon_{xc}(n, \vec{m})}{\partial m} \hat{m}$$

$B_{xc}$  and  $m$  are parallel

# Non-collinear case

$$H_P = -\frac{\hbar^2}{2m} \nabla^2 + V_{ef} + \mu_B \vec{\sigma} \cdot \vec{B}_{ef} + \zeta (\vec{\sigma} \cdot \vec{l}) \dots$$

- direction of magnetization vary in space
- spin-orbit coupling is present

$$\begin{pmatrix} -\frac{\hbar^2}{2m} \nabla^2 + V_{ef} + \mu_B B_z + \dots & \mu_B (B_x - iB_y) \\ \mu_B (B_x + iB_y) & -\frac{\hbar^2}{2m} \nabla^2 + V_{ef} + \mu_B B_z + \dots \end{pmatrix} \Psi = \epsilon \Psi$$

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}, \quad \Psi_1, \Psi_2 \neq 0$$

- solutions are **not pure spinors**
- **non-collinear** magnetic moments

# Collinear case

$$H_P = -\frac{\hbar^2}{2m} \nabla^2 + V_{ef} + \mu_B \vec{\sigma} \cdot \vec{B}_{ef} + \cancel{\zeta_s (\vec{\sigma} \cdot \vec{l})} \dots$$

- magnetization in Z direction,  $B_x$  and  $B_y=0$
- spin-orbit coupling is not present

$$\begin{pmatrix} -\frac{\hbar^2}{2m} \nabla^2 + V_{ef} + \mu_B B_z + \dots & 0 \\ 0 & -\frac{\hbar^2}{2m} \nabla^2 + V_{ef} + \mu_B B_z + \dots \end{pmatrix} \Psi = \varepsilon \Psi$$

$$\Psi_{\uparrow} = \begin{pmatrix} \Psi_1 \\ 0 \end{pmatrix}, \quad \Psi_{\downarrow} = \begin{pmatrix} 0 \\ \Psi_2 \end{pmatrix}, \quad \varepsilon_{\uparrow} \neq \varepsilon_{\downarrow}$$

- solutions are **pure spinors**
- **collinear** magnetic moments

# Non-magnetic case

$$H_P = -\frac{\hbar^2}{2m} \nabla^2 + V_{ef} + \cancel{\mu_B \vec{\sigma} \cdot \vec{B}_{ef}} + \cancel{\zeta (\vec{\sigma} \cdot \vec{l})} \dots$$

- no magnetization present,  $B_x, B_y$  and  $B_z=0$
- spin-orbit coupling is not present

$$\begin{pmatrix} -\frac{\hbar^2}{2m} \nabla^2 + V_{ef} + \dots & 0 \\ 0 & -\frac{\hbar^2}{2m} \nabla^2 + V_{ef} + \dots \end{pmatrix} \psi = \epsilon \psi$$

$$\psi_{\uparrow} = \begin{pmatrix} \psi \\ 0 \end{pmatrix}, \quad \psi_{\downarrow} = \begin{pmatrix} 0 \\ \psi \end{pmatrix}, \quad \epsilon_{\uparrow} = \epsilon_{\downarrow}$$

- solutions are **pure spinors**
- **degenerate spin solutions**

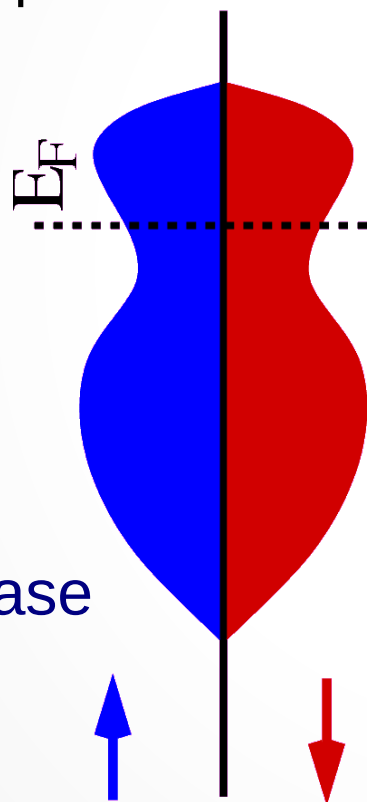


# Magnetism and Wien2k

- Wien2k can only handle **collinear or non-magnetic** cases

*run\_lapw* script: DOS

```
x lapw0
x lapw1
x lapw2
x lcore
x mixer
```

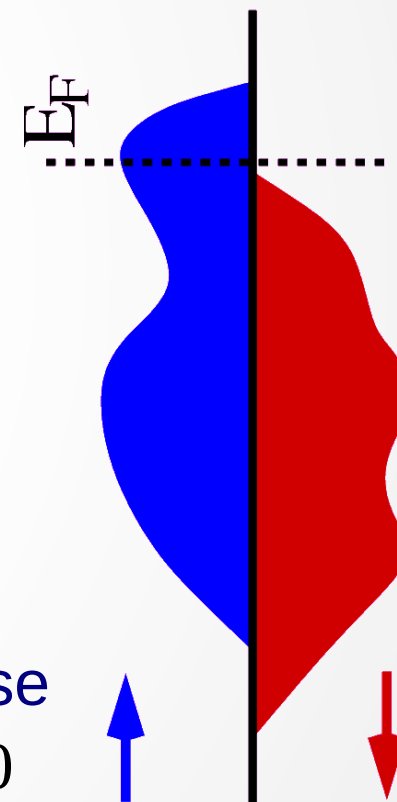


non-magnetic case

$$m = n_{\uparrow} - n_{\downarrow} = 0$$

*runsp\_lapw* script: DOS

```
x lapw0
x lapw1 -up
x lapw1 -dn
x lapw2 -up
x lapw2 -dn
x lcore -up
x lcore -dn
x mixer
```



magnetic case

$$m = n_{\uparrow} - n_{\downarrow} \neq 0$$

# Magnetism and Wien2k

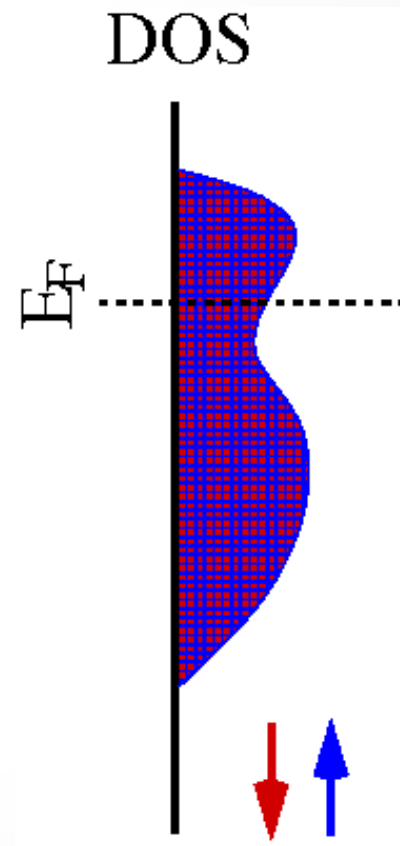
- in NCM case both part of the spinor are treated simultaneously

$$\hat{n} = \sum_{nk} \begin{pmatrix} \Psi_{\uparrow nk} \\ \Psi_{\downarrow nk} \end{pmatrix}^* \begin{pmatrix} \Psi_{\uparrow nk} & \Psi_{\downarrow nk} \end{pmatrix}$$

$$m_z = n_{\uparrow\uparrow} - n_{\downarrow\downarrow} \neq 0$$

$$m_x = \frac{1}{2}(n_{\uparrow\downarrow} + n_{\downarrow\uparrow}) \neq 0$$

$$m_y = i \frac{1}{2}(n_{\uparrow\downarrow} - n_{\downarrow\uparrow}) \neq 0$$



# Non-collinear calculations

- **in the case of non-collinear arrangement of spin moment **WienNCM** (Wien2k clone) has to be used**
  - code is based on Wien2k (available for Wien2k users)
  - structure and usage philosophy similar to Wien2k
  - independent source tree, independent installation
- **WienNCM properties:**
  - real and **spin symmetry** (simplifies SCF, less k-points)
  - constrained or unconstrained calculations (optimizes magnetic moments)
  - SOC is applied in the first variational step, LDA+U

# WienNCM - implementation

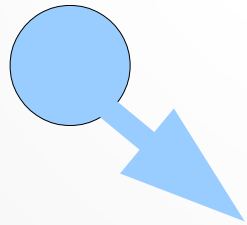
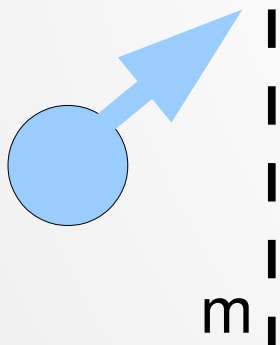
- **basis set – mixed spinors** (Yamagami, PRB (2000); Kurtz PRB (2001))

interstices:  $\varphi_{\vec{G}\sigma} = e^{i(\vec{G}+\vec{k})\cdot\vec{r}} \chi_{\sigma} \quad \chi_{\sigma} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

spheres:  $\varphi_{\vec{G}\sigma}^{APW} = \sum_{\sigma_{\alpha}} \sum_{lm} \left( A_{lm}^{\vec{G}\sigma\sigma_{\alpha}} u_l^{\sigma_{\alpha}} + B_{lm}^{\vec{G}\sigma\sigma_{\alpha}} \dot{u}_l^{\sigma_{\alpha}} \right) Y_{lm} \chi_{\sigma_{\alpha}}$

$\varphi_{\vec{G}\sigma_{\alpha}}^{APW} = \left( A_{lm}^{\vec{G}\sigma\sigma_{\alpha}} u_l^{\sigma_{\alpha}} + B_{lm}^{\vec{G}\sigma\sigma_{\alpha}} \dot{u}_l^{\sigma_{\alpha}} + C_{lm}^{\vec{G}\sigma\sigma_{\alpha}} u_{2,l}^{\sigma_{\alpha}} \right) Y_{lm} \chi_{\sigma_{\alpha}}$

- **real and spin space parts of symmetry op. are not independent**



- symmetry treatment like for SOC always on
- tool for setting up magnetic configuration
- concept of magnetic and non-magnetic atoms

# WienNCM implementation

- Hamiltonian inside spheres:

$$\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + \hat{V} + \hat{H}_{so} + \hat{H}_{orb} + \hat{H}_c$$

AMA and full NC calculation

$$\hat{V}_{FULL} = \begin{pmatrix} V_{\uparrow\uparrow} & V_{\downarrow\uparrow} \\ V_{\uparrow\downarrow} & V_{\downarrow\downarrow} \end{pmatrix} \quad \hat{V}_{AMA} = \begin{pmatrix} V_{\uparrow\uparrow} & 0 \\ 0 & V_{\downarrow\downarrow} \end{pmatrix}$$

SOC in first diagonalization

$$\hat{H}_{so} = \xi \vec{\sigma} \cdot \vec{l} = \xi \begin{pmatrix} \hat{l}_z & \hat{l}_x - i\hat{l}_y \\ \hat{l}_x + i\hat{l}_y & -\hat{l}_z \end{pmatrix}$$

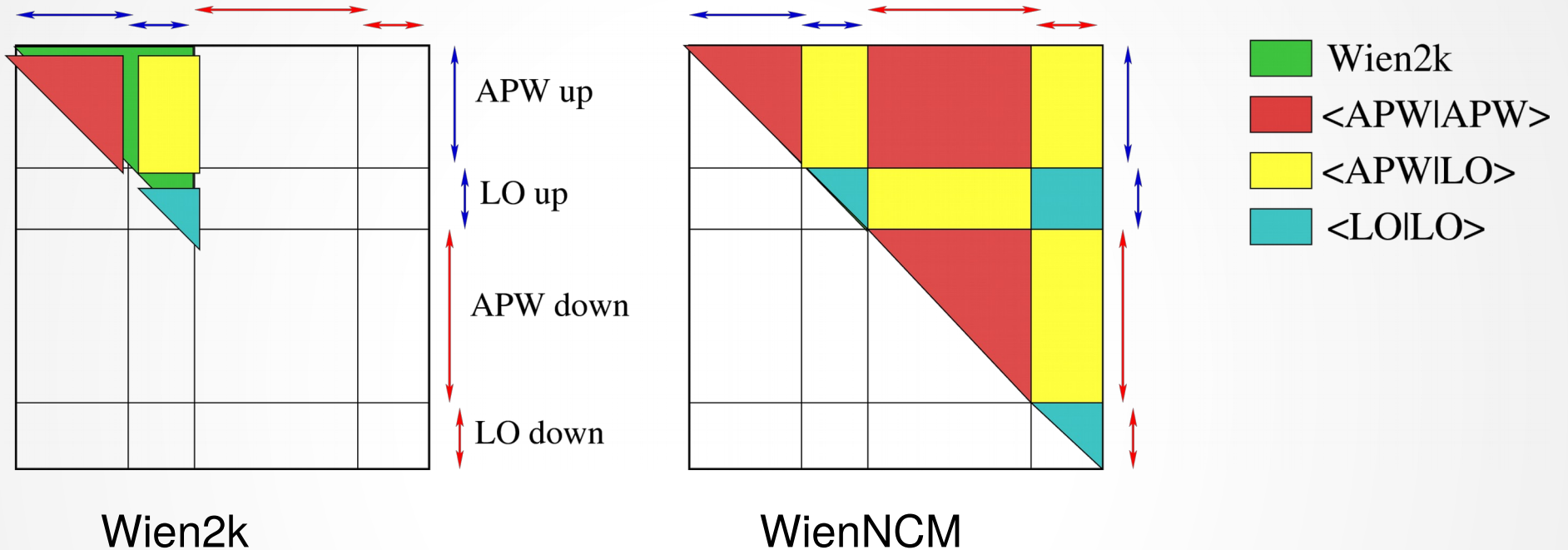
diagonal orbital field

$$\hat{H}_{orb} = \sum_{mm'} \begin{pmatrix} |m\rangle V_{mm'}^{\uparrow} \langle m'| & 0 \\ 0 & |m\rangle V_{mm'}^{\downarrow} \langle m'| \end{pmatrix}$$

constraining field

$$\hat{H}_c = \mu_B \vec{\sigma} \cdot \vec{B}_c = \begin{pmatrix} 0 & \mu_B (B_{cx} - iB_{cy}) \\ \mu_B (B_{cx} + iB_{cy}) & 0 \end{pmatrix}$$

# NCM Hamiltonian

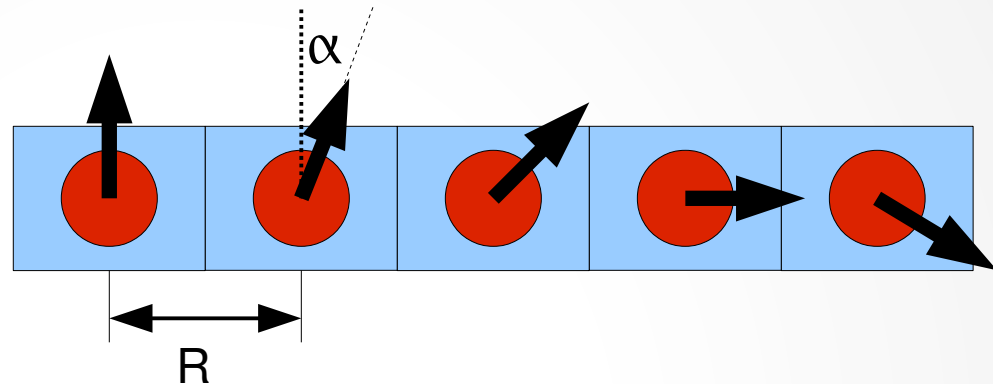


- size of the Hamiltonian/overlap matrix is doubled comparing to Wien2k
- computational cost increases !!!

# WienNCM – spin spirals

- transverse spin wave

$$\alpha = \vec{R} \cdot \vec{q}$$



$$\vec{m}^n = m \left( \cos(\vec{q} \cdot \vec{R}^n), \sin(\vec{q} \cdot \vec{R}^n) \sin(\theta), \cos(\theta) \right)$$

- spin-spiral is defined by a vector  $\mathbf{q}$  given in reciprocal space and,
- an angle  $\Theta$  between magnetic moment and rotation axis
- rotation axis is arbitrary (no SOC), hard-coded as Z

**Translational symmetry is lost !!!**

# WienNCM – spin spirals

- generalized Bloch theorem
  - generalized translations are symmetry operation of the H

$$T_n = \left\{ -\vec{q} \cdot \vec{R}_n \mid \epsilon \mid \vec{R}_n \right\}$$

$$T_n^\dagger H(\vec{r}) T_n = U^\dagger(-\vec{q} \cdot \vec{R}_n) H(\vec{r} + \vec{R}_n) U(-\vec{q} \cdot \vec{R}_n)$$

group of  $T_n$  is Abelian

$$\psi_{\vec{k}}(\vec{r}) = e^{i(\vec{k} \cdot \vec{r})} \begin{pmatrix} e^{\frac{i\vec{q} \cdot \vec{r}}{2}} u^\uparrow(\vec{r}) \\ e^{-\frac{i\vec{q} \cdot \vec{r}}{2}} u^\downarrow(\vec{r}) \end{pmatrix}$$

$$T_n \psi_{\vec{k}}(\vec{r}) = U(-\vec{q} \cdot \vec{R}) \psi_{\vec{k}}(\vec{r} + \vec{R}) = e^{i\vec{k} \cdot \vec{r}} \psi_{\vec{k}}(\vec{r})$$

1-d representations,  
Bloch Theorem

- efficient way for calculation of spin waves, only one unit cell is necessary for even incommensurate wave



# Usage

- generate atomic and magnetic structure

- 1) create atomic structure
- 2) create magnetic structure

need to specify only directions of magnetic atoms

use utility programs: **ncmsymmetry**, **polarangles**, ...

- run **initncm** (initialization script)
- **xncm** ( WienNCM version of **x** script)
- **runncm** (WienNCM version of **run** script)
- find more in manual

# WienNCM – case.inncm file

- case.inncm – magnetic structure file

FULL		
0.000	0.000	0.000
45.00000	54.73561	0
135.00000	125.26439	0
-135.00000	54.73561	0
-45.00000	125.26439	0
45.00000	54.73561	0
45.00000	54.73561	0
315.00000	125.26439	0
315.00000	125.26439	0
135.00000	125.26439	0
135.00000	125.26439	0
225.00000	54.73561	0
225.00000	54.73561	0
0.50000		

q spiral vector

polar angles of mm

optimization switch

U, magnetic atoms

O, non-magnetic atoms

mixing for constraining field

The diagram shows a 3D crystal structure with U (magnetic) atoms in teal and O (non-magnetic) atoms in pink. Red arrows represent magnetic moments on the U atoms. The structure is a complex lattice with various coordination environments. Labels 0.1 through 0.12 are placed near the O atoms.

# how to run it ?

- similar to WIEN2k (*initncm*, *runncm*, *xncm* ...)

```
runncm_lapw -p -cc 0.0001 ...
```

```
xncm lapw0
```

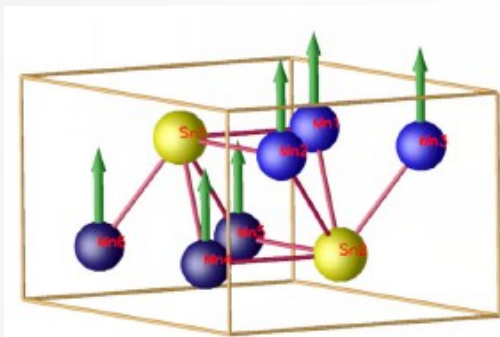
```
xncm lapw1
```

```
xncm lapw2
```

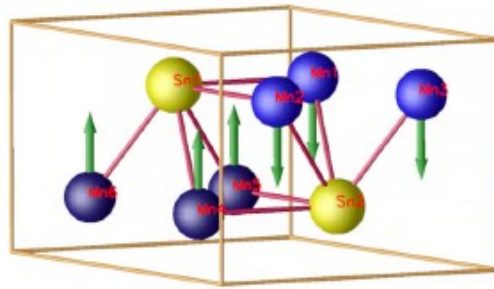
```
xncm lcore
```

```
xncm mixer
```

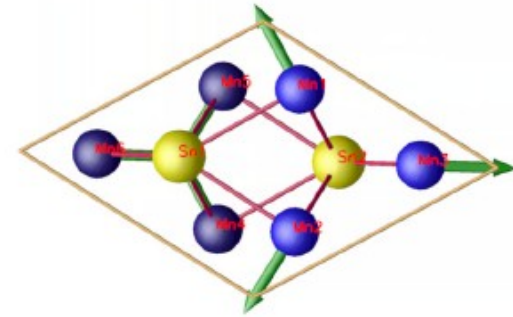
# Magnetic structure of $\text{Mn}_3\text{Sn}$



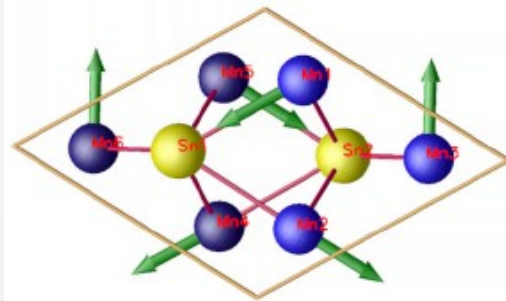
fm



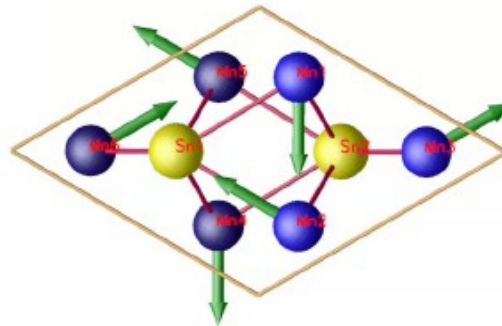
afm



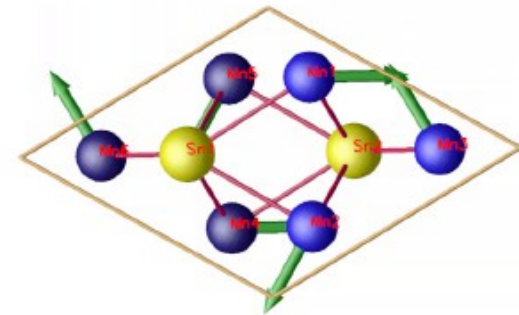
ncm 1



ncm 2



ncm 3

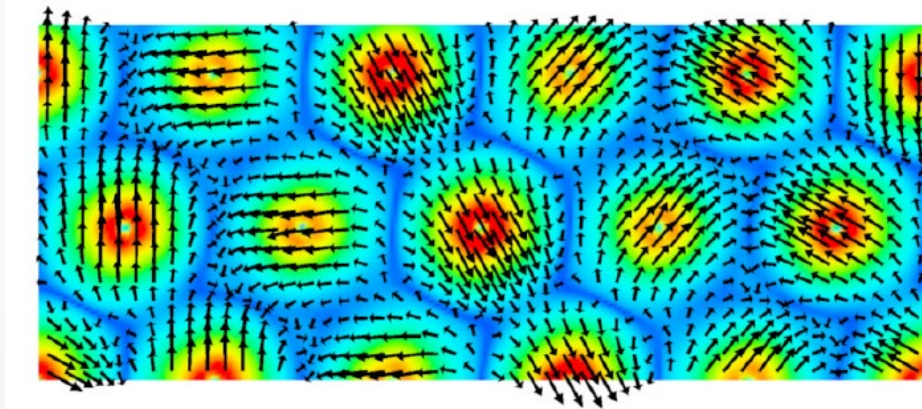
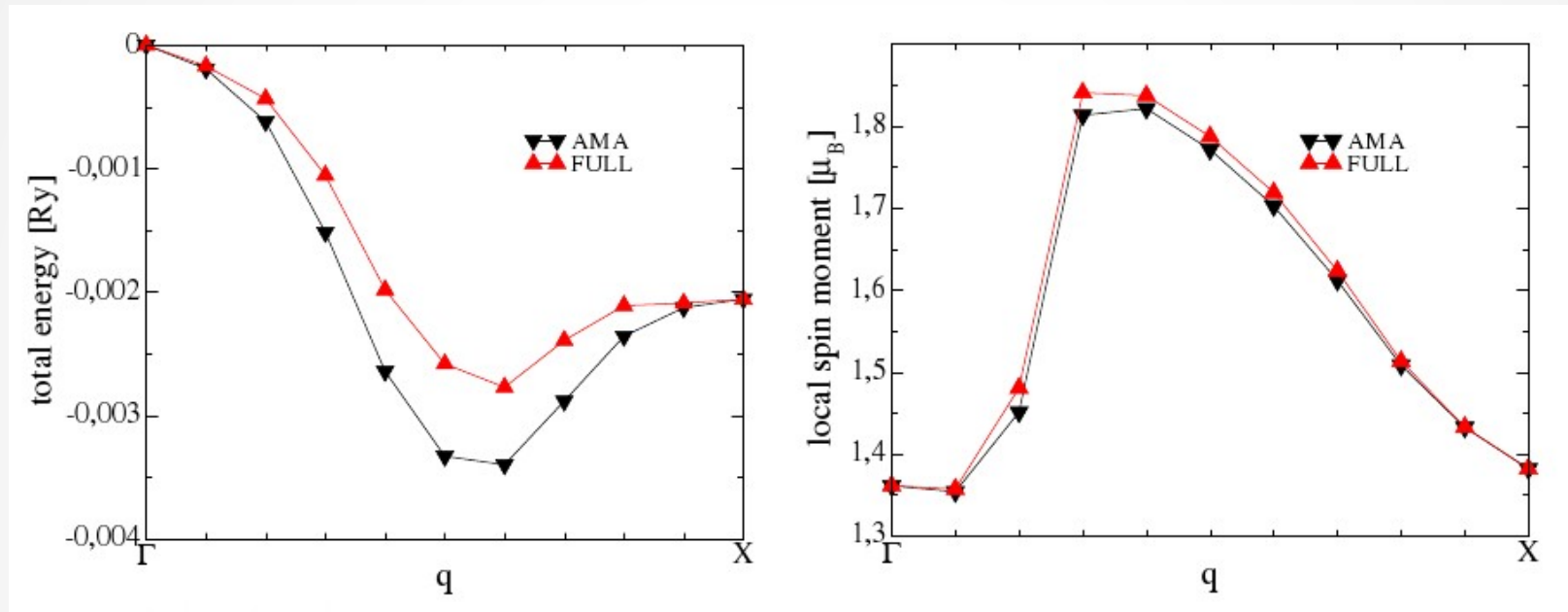


ncm 4

# Mn<sub>3</sub>Sn cd.

	so	fm	afm	ncm 1	ncm 2	ncm 3	ncm 4
$E_{fm} - E$ [Ry]	-	0.0	0.0131	0.0444	0.0444	0.0444	0.0444
	+	0.0	0.0133	0.0441	0.0439	0.0444	0.0445
$M_s$ [ $\mu_B$ ]	-	3.012	2.684	3.037	3.037	3.037	3.037
	+	3.008	2.679	3.034	3.034	3.038	3.037
$efg$ on Mn [ $10^{21}V/m^2$ ]	-	-1.657	-2.111	-0.894	-0.894	-0.894	-0.894
	+	-1.661	-2.119	-0.892	-0.899	-0.891	-0.894
						-0.898	-0.881
$hff$ on Mn [kGauss]	-	-309.9	-153.1	31.2	31.2	31.2	31.2
	+	-309.6	-152.9	31.1	31.5	31.5	30.9
						32.2	32.1

# $\gamma$ Fe, spin spiral



Spin density maps for  $q = 0.6$  ( $0-\Gamma, 1-X$ )